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SHORTER COMMUNICATION

FURTHER NOTE ON AN APPROXIMATE METHOD FOR CALCULATING HEAT TRANSFER IN LAMINAR BOUNDARY-LAYERS WITH CONSTANT WALL TEMPERATURE

A. G. SMITH* and V. L. SHAH†

NOMENCLATURE

- c, characteristic length of body, e.g. chord or major axis, ft;
- c_p, specific heat at constant pressure, CHU lb^{-1} $degC^{-1}$;
- Δ_4, conduction thickness of the thermal boundary layer defined by $h = k/\Delta_4$, ft;
- h, coefficient of heat transfer, CHU ft^{-2} $degC^{-1}$ sec^{-1};
- k, thermal conductivity of the fluid, CHU ft^{-1} $degC^{-1}$;
- Nu, Nusselt number hc/k;
- ν, kinematic viscosity of the fluid, ft^2 s^{-1};
- Pr, Prandtl number, $\rho c_p \nu/k$;
- Re_c, Reynolds number, Uc/ν;
- ρ, fluid density, lb ft^{-3};
- U_1, mainstream velocity at a point on the surface, ft s^{-1};
- U, approach or reference velocity, ft s^{-1};
- x, distance along surface from stagnation point, ft.

IN A PREVIOUS paper [1] the method of Smith and Spalding [2], for the calculation of heat transfer in laminar boundary-layers with constant wall temperature, was extended to the range of Prandtl numbers 0·7–10.

The present note extends the previous paper in two respects:

(a) It compares the results of calculations by the method of [1], with other theoretical results, and with some experimental measurements at $Pr = 2{\cdot}5$.

(b) As a result of the data of Evans [3] it refines the values of the constants in the quadratures of [1] and extends the range of Prandtl numbers to $0{\cdot}5 < Pr < 20{,}000$.

The theoretical results with which the comparison is made are those of Merk [4] whose method is based on the exact "wedge" solutions of Eckert [5]. Figs. 1 and 2 show $Nu/\sqrt{Re_c}$ calculated by the two methods for flow over ellipses of 2 : 1 and 4 : 1 major/minor axis ratio. The experimental results with which comparison is made are those of Sogin *et al.* [6], who measured mass transfer from cylinders in cross flow. Comparisons are shown in Figs. 3–5. Agreement is satisfactory.

It is confirmed, therefore, that our method described in [1] predicts heat transfer fairly accurately except near separation, where it predicts rather high values.

Evans [3] has greatly extended and refined the "wedge-flow" exact solutions of the thermal boundary-layer equations. From his data, the values of A and B, which are the constants in the quadrature given in paper [1],

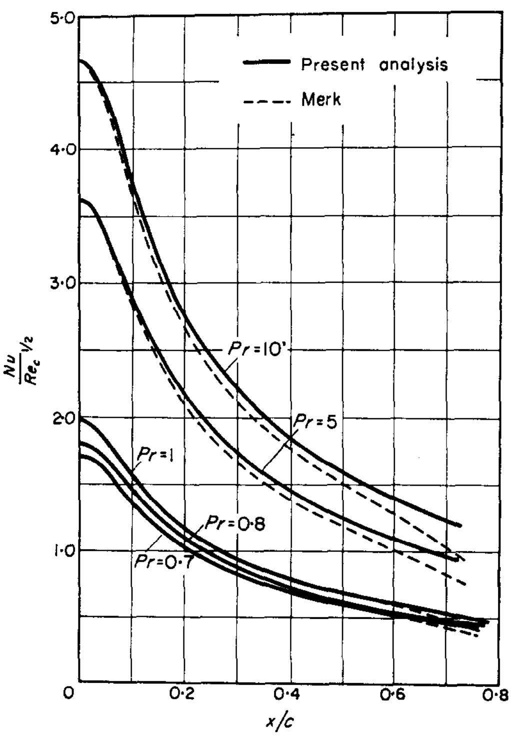

FIG. 1. Heat-transfer coefficient on the surface of a 2 : 1 ellipse for various Prandtl numbers.

* University of Nottingham, Nottingham, United Kingdom.

† The College of Aeronautics, United Kingdom.

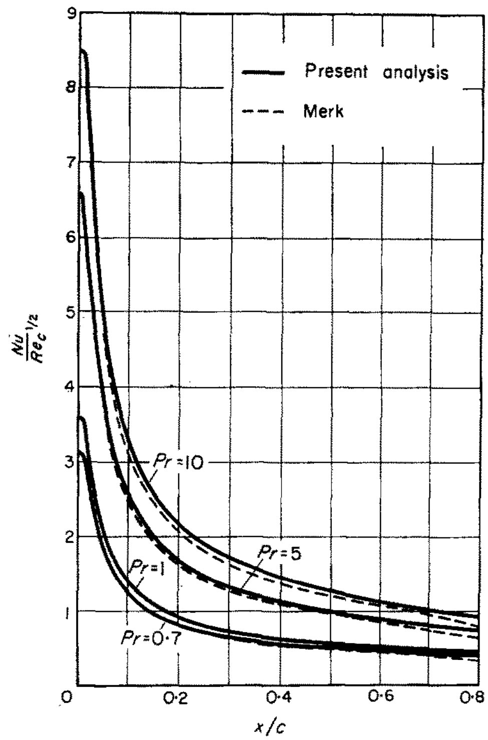

FIG. 2. Heat-transfer coefficient on the surface of a 4 : 1 ellipse for various Prandtl numbers.

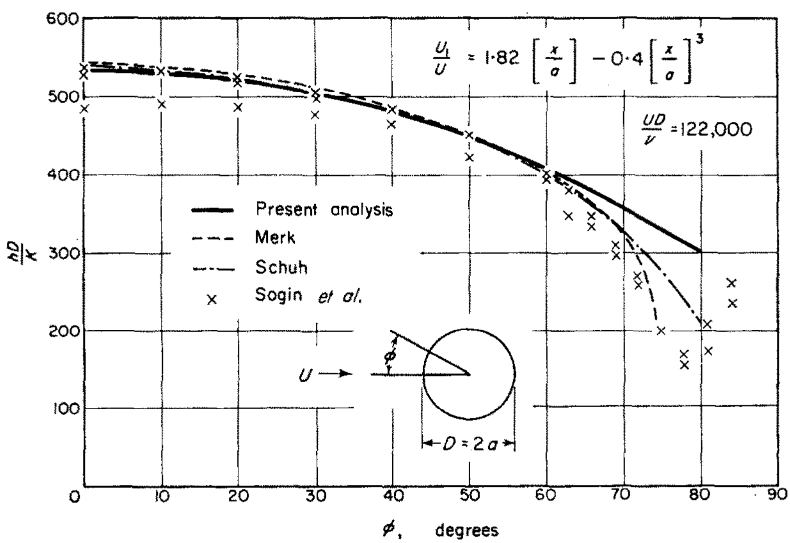

FIG. 3. Local rates of heat transfer from cylinders in cross flow, $Pr = 2{\cdot}5$.

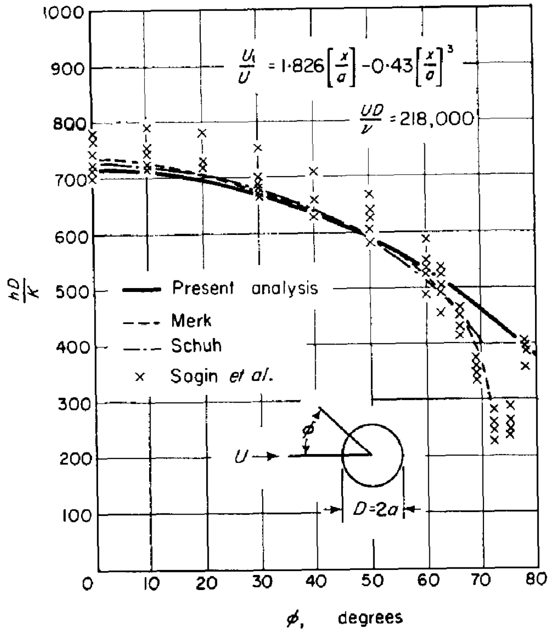

FIG. 4. Local rates of heat transfer from cylinders in cross flow, $Pr = 2{\cdot}5$.

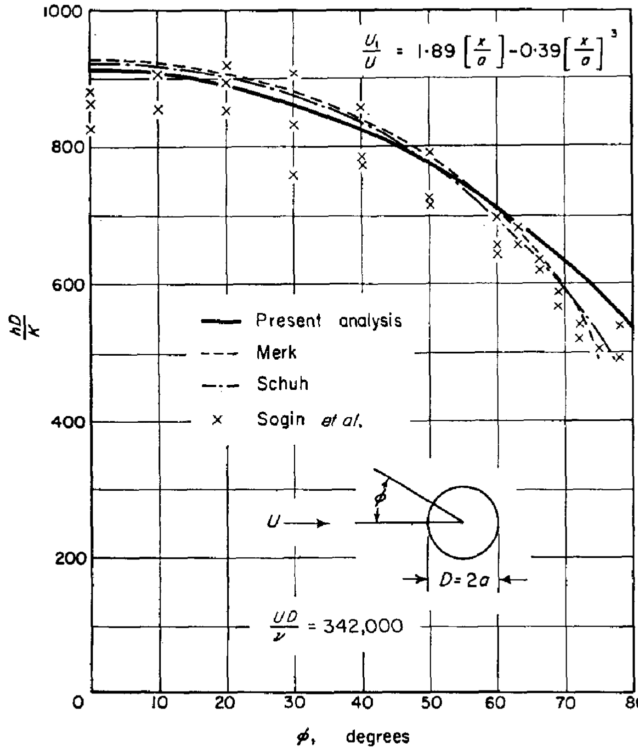

FIG. 5. Local rates of heat transfer from cylinders in cross flow, $Pr = 2{\cdot}5$.

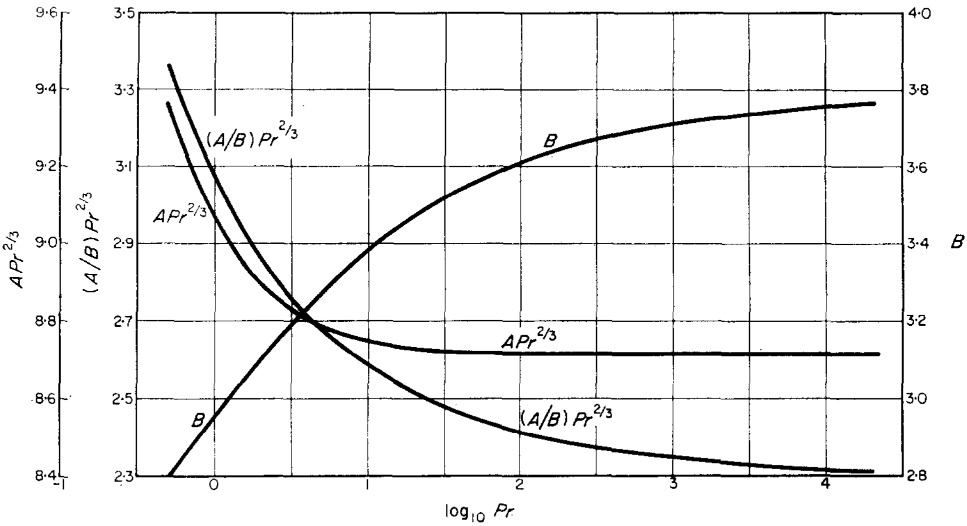

FIG. 6. Graphs of dependence of numbers A and B on Pr.

have been calculated, and are presented in Table 1 and Fig. 6. The constants A and B are used in the quadrature

$$\left(\frac{\Delta_4}{c}\right)^2\left(\frac{\nu}{Uc}\right) = \frac{A}{(U_1/U)^B}\int_0^{x/c}\left(\frac{U_1}{U}\right)^{B-1} d\left(\frac{x}{c}\right) + \frac{1}{(U_1/U)^B}\int_0^{x/c}\left(\frac{U_1}{U}\right)^{B-1} E_4\, d\left(\frac{x}{c}\right)$$

E_4, which is a function of

$$\frac{\Delta_4^2}{\nu}\frac{dU_1}{dx},$$

is given in [1] for $0{\cdot}7 < Pr < 10$. For Pr values greater than 10, it will often be satisfactory, when it is desired to include the error term in the calculation, to assume similarity of the E_4 values with those for $Pr = 10$. By this is meant

$$\frac{E_4}{A} = f\left[\left(\frac{\Delta_4^2}{\nu}\frac{dU_1}{dx}\right)/D\right],$$

where D is the interval of $(\Delta_4^2/\nu)(dU_1/dx)$ between the values at which $E_4 = 0$.

Table 1

Pr	$APr^{2/3}$	B	$(A/B)Pr^{2/3}$
0·5	9·369	2·791	3·357
0·7	9·204	2·869	3·207
0·8	9·146	2·900	3·154
1·0	9·069	2·952	3·072
1·4	8·975	3·026	2·966
2	8·902	3·102	2·870
3	8·840	3·181	2·779
5	8·791	3·273	2·686
10	8·754	3·380	2·590
50	8·724	3·558	2·452
100	8·712	3·606	2·416
500	8·717	3·692	2·361
1000	8·716	3·716	2·346
5000	8·716	3·753	2·322
10 000	8·716	3·764	2·316
20 000	8·716	3·773	2·310

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